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# GEOMETRY.

106. Proposed by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Upon the sides of any triangle  $ABC$  let the equilateral triangles  $ABD$ ,  $BCE$ , and  $CAF$  be described, and let their exterior sides produced intersect  $BE$  and  $AF$  in  $K$ ,  $DB$  and  $FC$  in  $L$ , and  $DA$  and  $EC$  in  $M$ . Prove  $DK$ ,  $EL$ ,  $FM$ , parallel.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and G. I. HOPKINS, Instructor in Mathematics and Physics, High School, Manchester, N. H.

Angle  $KAB = 180^\circ - 60^\circ - \text{angle } CAB$ , angle  $ABK = 180^\circ - 60^\circ - \text{angle } ABC$ .

$\therefore$  Adding, angle  $KAB + \text{angle } ABK = 240^\circ - \text{angle } CAB - \text{angle } ABC$ . Angle  $AKB = 180^\circ - (\text{angle } KAB + \text{angle } ABK)$ .

$\therefore$  Angle  $AKB = \text{angle } CAB + \text{angle } ABC - 60^\circ$ .

Again, angle  $BCL = 180^\circ - 60^\circ - \text{angle } ACB$ . Angle  $ACB = 180^\circ - (\text{angle } CAB + \text{angle } ABC)$ .

$\therefore$  Angle  $BCL = \text{angle } CAB + \text{angle } ABC - 60^\circ$ .

$\therefore$  Angle  $BCL = \text{angle } AKB$ .

Angle  $MAC = 60^\circ + \text{angle } MAF$ , and angle  $KAB = 60^\circ + \text{angle } KAD$ .  $\therefore$  Angle  $MAC = \text{angle } KAB$ .

Similarly, angle  $MCA = \text{angle } BCL$ .  $\therefore$  Angle  $MCA = \text{angle } AKB$ .

$\therefore$  Triangle  $AMC$  is similar to triangle  $AKB$ .

$\therefore AC : AK :: AM : AB$ , or  $AF : AK :: AM : AD$ .

$\therefore$  Triangle  $AKF$  is similar to triangle  $AKD$ .

$\therefore$  Angle  $AMF = \text{angle } ADK$ .

$\therefore KD$  is parallel to  $MF$ . Similarly  $EL$  is parallel to  $MF$ .

II. Solution by the PROPOSER.

Points  $K, F, C, B$  are concyclic.  $\therefore \angle CKE = \angle AFC$ .

$\therefore \angle BKA = \angle BCL$ .  $\angle LBC = \angle KBA$ .

$\therefore$  Triangles  $AKB$  and  $CBL$  are similar, and  $KB/AB = LB/CB$ , or  $KB/EB = LB/EB$ .

$\therefore KD$  is parallel to  $EL$ .

Similarly  $AM/AD = AD/AK$ , and therefore  $FD$  is parallel to  $KD$ .

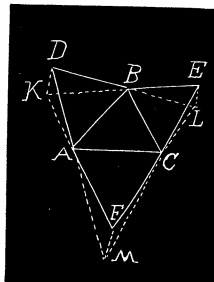
And therefore  $EL$  is parallel to  $KD$  is parallel to  $FM$ .

III. Solution by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.

Let  $ABC$  be any triangle having equilateral triangles described upon its sides, and their exterior sides produced to intersect  $BE$  and  $AF$  in  $K$ ,  $DB$  and  $FC$  in  $L$ , and  $DA$  and  $EC$  in  $M$ . Join  $FM$ ,  $DK$  and  $EL$ .

The triangles  $BCL$  and  $ACM$  are similar, hence  $BC : CL :: CM : CA$ , or  $CE : CL :: CM : CF$ . And since the  $\angle ECL = \angle FCM$ , the triangles  $CLE$  and  $CFM$  are similar and equiangular, the angle  $FMC$  being equal to the angle  $LEC$ .

$\therefore ER$  is parallel to  $FM$ .....(1).



The triangles  $ABK$  and  $AMC$  are similar, hence  $AB : AK :: AM : AC$ , or  $AD : AK :: AM : AF$ .

Since the  $\angle DAK = \angle MAF$ , the triangles  $DKA$  and  $MFA$  are similar, and  $\angle ADK$  is equal to  $\angle AMF$ .

$\therefore DH$  is parallel to  $FM$ ..... (2).

$\therefore DK, FM$  and  $EL$  are parallel. Q. E. D.

#### IV. Solution by CHARLES C. CROSS, Libertytown, Md.

Draw the figure as indicated in the problem.

Let  $\angle BLE = x$ ,  $\angle CEL = y$ ,  $\angle DKB = z$ ,  $\angle ADK = w$ ,  $\angle CEM = v$ , and  $\angle FMC = w$ .

$$\angle ECL = 180^\circ - (120^\circ + C) = 60^\circ - C.$$

Similarly,  $\angle EBL = A - 60^\circ$ , and  $\angle KAD = 60^\circ - A$ .

$$\angle BCL = 180^\circ - (60^\circ + C) = 120^\circ - C.$$

Similarly,  $\angle LBC = 120^\circ - B$ , and  $\angle BAK = 120^\circ - A$ .

Hence  $\angle BLC = B + C - 60^\circ$ , and  $\angle BKA = B + A - 60^\circ$ .

$\angle BLE + \angle BLC + \angle CEL + \angle ECL = 180^\circ$ ; by substitution  $B + x + y = 180^\circ$ ... (1).

$\angle BKA + \angle BKD + \angle KDA + \angle KAD = 180^\circ$ ; by substitution  $B + w + z = 180^\circ$ ... (2).

From (1) and (2),  $x + y = w + z$ ..... (3).

If  $EL$  and  $DK$  are parallel, angle  $DKB = \text{angle } BEL$ , and angle  $BLE = \text{angle } KDB$ , or  $z = 60^\circ + y$  and  $x = 60^\circ + w$ . Substituting in (3),  $60^\circ + w + y = 60^\circ + w + y$ . Hence  $EL$  and  $DK$  are parallel.

Angle  $CFM + \text{angle } CMF + \text{angle } FCL = 180^\circ$ ; by substitut'n  $v + w - C = 120^\circ$ ... (4).

If  $EL$  and  $FM$  are parallel, then angle  $MFC = \text{angle } ELC$ , and angle  $EMC = \text{angle } CEL$ , or  $v = x + A + C - 60^\circ$ , and  $w = y$ . Substituting in (4),  $A + x + y = 180^\circ$ . Since by (1) this relation is true, hence  $EL$  and  $FM$  are parallel.

#### 107. Proposed by T. W. PALMER, A. M., Professor of Mathematics, University of Alabama.

Construct a triangle, given base, vertical angle and radius of inscribed circle.

#### I. Solution by H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Denote the base by  $AB$ , the vertex by  $C$ , and the incenter by  $O$ . The angle  $AOB$  equals  $90^\circ + \frac{1}{2}C$  and hence one locus for  $O$  is the arc of a segment capable of containing this angle. Another locus is a parallel to the base the in-radius away. Hence the incircle can be constructed;  $AC$  and  $BC$  are then drawn tangent to it.

#### II. Solution by J. SCHEFFER, A. M., Hagersfown, Md.

Describe on the given base  $AB$  a circle the upper segment of which contains the given vertical angle. From the center  $O$  of this circle let fall the perpendicular on  $AB$  and produce it to  $D$ . At a distance from  $AB$  equal to the given radius of the inscribed circle draw  $MN$  parallel to  $AB$ . From  $D$  as a center with a radius equal to  $BD$  draw an arc cutting  $MN$  at  $E$ , connect  $E$  with  $D$  and extend  $DE$  until it

